

Detecting long and short memory via spectral methods

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Abstract

We study the properties of memory of a financial time series adopting two different methods of analysis, the detrended fluctuation analysis (DFA) and the analysis of the power spectrum (PSA). The methods are applied on three time series: one of high-frequency returns, one of shuffled returns and one of absolute values of returns. We prove that both DFA and PSA give results in line with those obtained with standard econometrics measures of correlation.

Key words: serial correlation, high-frequency data, DFA, power spectrum, short memory, long memory.

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1 Introduction

The analysis of financial time series has become an important field of application for physicists. A special role plays the analysis of serial correlation in the time series, as it gives useful insight to the price formation mechanism. The study of serial correlation is not new, both in Econometrics and Econophysics. The presence of serial correlation in the time series of log-returns is forbidden by the efficient market hypothesis in its weak form, see [1]. At daily level the presence of these fluctuations has been found in several markets and interpreted in the rational framework, with non-synchronous trading [2] or institutional factors [3], or invoking behavioral factors [4].

The number of works on serial correlation with intraday and high-frequency data is, at our knowledge, very limited due mainly to the presence of microstructure effects in the time series that make difficult a direct analysis of the problem. Recent examples are Refs. [5] on the Italian futures index, [6] on the exchange rates and [7] on the Indian market. In this paper we show how the adoption of popular methods of statistical data analysis to infer about the presence of serial correlations at intraday level, can be meaningful interpreted at the light of econometrics measures, if the microstructure of the market is correctly taken into account. Particularly, we adopt a simple analysis of the power spectrum (PSA) associated with the signal, and the famous Detrended Fluctuation Analysis (DFA) method.

The paper is organized as follows: Sections 2 and 3 introduce the statistical instruments used, the data set is described in Section 4, Section 5 shows the methodology that we employ and discuss the results at the light of recent works on the same topic, while Section 6 concludes.

2 Power spectrum analysis

Let $x(t)$ be a stochastic process, we define the power spectrum as the square modulus of the Fourier transform of the signal, namely:

$$G(f) = |\tilde{x}(f)|^2. \quad (1)$$

The power spectrum is linked to the autocorrelation function of the signal by the Wiener-Khintchine theorem, namely by the following equation:

$$\rho(\tau) = \int_0^{\infty} G(f) \cos(2\pi f \tau) df \quad (2)$$

which admits the following inverse relation:

$$G(f) = 4 \int_0^{\infty} \rho(\tau) \cos(2\pi f \tau) d\tau. \quad (3)$$

In absence of correlations, the correlation function is δ -peaked and the power spectrum is flat (white noise). If on the contrary there is serial correlation, this does not hold anymore and therefore we observe a decay in the power spectrum with the frequency as follows:

$$G(f) \sim f^{-\eta} \quad (4)$$

By monitoring the decay of $G(f)$ we can infer about the memory properties of the time series under study. To perform the PSA we use a fast Fourier transform algorithm.

3 Detrended Fluctuation Analysis

The method of DFA has been widely adopted in the literature, starting from the analysis of DNA sequences [8] to financial time series [9], from ecological applications [10] to nuclear reactions related problems [11]. It consists on the evaluation of the scaling properties of the locally detrended standard deviation of the time series. We shall now briefly introduce the algorithm.

Let again x_i , ($i = 1, \dots, N$) be a stochastic process. The method consists on the following steps:

- build the integrated time series $y_k = \sum_{i=1}^k (x_i - \bar{x})$, ($k = 1, \dots, N$) where \bar{x} is the mean value of the signal;
- divide the new sequence in $n = N/l$ non-overlapping subsequences of length l ;
- evaluate the local trend of each subsequence, $\bar{y}_k^l(n)$;
- evaluate the summation of the differences between the integrated time series and the local trend in the time window l , and take its standard deviation, t. i. the quantity:

$$F(l) = \sqrt{\frac{1}{N} \sum_{k=1}^N [y_k - \bar{y}_k^l(n)]^2}. \quad (5)$$

The modified standard deviation $F(l)$ so built has the following scaling property:

$$F(l) \sim l^\alpha. \quad (6)$$

If $\alpha = 0.5$ the process is a white noise and there no autocorrelation; if $0.5 < \alpha < 1$, then there is significant autocorrelation in the time series. In the next Section we shall describe the data set we use for the analysis of this paper.

4 Data set description

The data set at our disposal consists of all the transactions taken from the Italian futures on the stock index S&PMIB, named FIB30, in the period from January 2000 to December 2002. We use only the next-to-expiration contracts. The data are evenly spaced according to a previous interpolation procedure at a time lag of 1 minute. The spacing has the effect of getting rid of microstructure effects that can spoil the analysis, see Ref. [5] for a complete discussion. We have 751 trading days, for a total of 8,657,949 transactions. After the spacing procedure, we have 495 1-minute returns per day. The same data set has been also used in recent publications, see Refs. [12,?].

In the next Section we shall see how the analysis is performed.

5 Methodology and results

In this Section we describe the methodology that we use to evaluate the memory properties of the time series. We apply the methods introduced in the previous Sections to three time series: one made of high-frequency returns, whose memory properties are unknown, one obtained by shuffling the returns and therefore without memory, and one of absolute returns, which are known to be long range autocorrelated. We repeat the procedure for every day of our data set and build the distribution of the coefficients obtained from the PSA and the DFA. We then plot the three distributions and compare them to infer the memory characteristics of the original time series. The results are in Figs. 1 and 2.

It is evident from the Figures that both the methods agree in detecting no

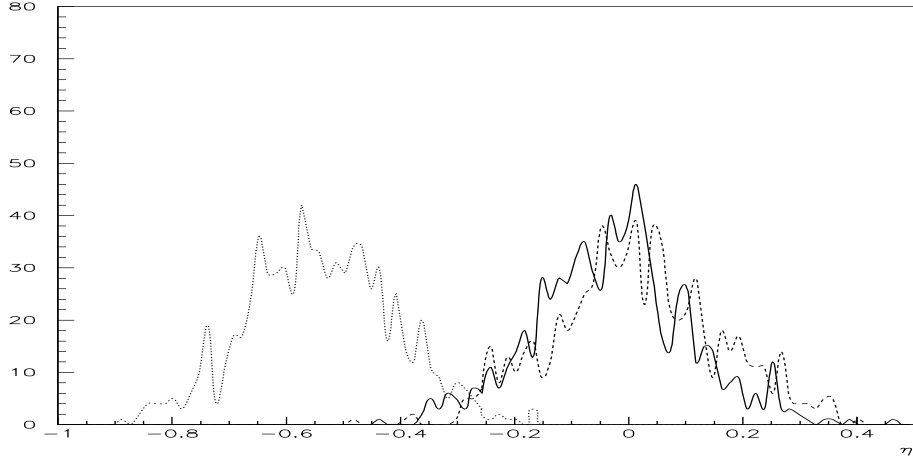


Fig. 1. In this Figure the distribution of the decay coefficients η obtained from the PSA algorithm on three time series: a time series of absolute returns (dotted line), a time series of shuffled returns (dashed line) and the original time series of returns (solid line).

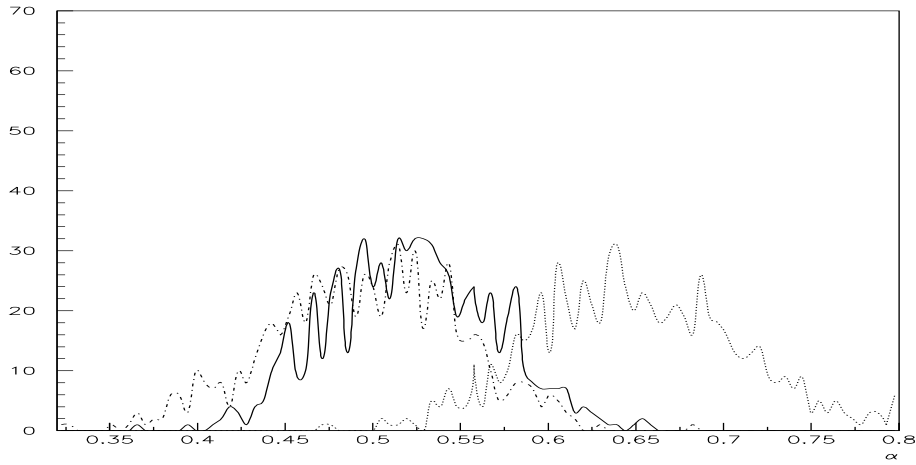


Fig. 2. In this Figure the distribution of the scaling coefficients α obtained from the application of DFA on three time series: a time series of absolute returns (dotted line), a time series of shuffled returns (dashed line) and the original time series of returns (solid line).

memory in the time series of shuffled returns: both distributions are centered around the expected value, that is 0 for PSA and 0.5 for DFA. We note that the standard deviation of the distributions is a reliable measure of the statistical uncertainty of the coefficient. Moreover the results of the two methods when

applied to the time series of absolute returns are, as expected, compatible with the hypothesis of long memory of this time series. When applied to the time series of real returns, both DFA and PSA suggest the presence of short memory effects in the time series, being the distributions of coefficients between the other two.

In order to properly address the results we refer again to Ref. [5], where the authors were able to prove, adopting only econometric indicators, the presence of short memory in the time series of returns. Our study therefore suggests that the adoption of DFA and PSA is effective in the evaluation of short and long term memory in financial time series.

6 Concluding remarks

In this paper we prove that an effective relation exists between what is expected adopting scaling method of analysis, as DFA, spectral methods, as PSA, and econometric indicators, as in Ref. [5] in the evaluation of intraday serial correlations in high-frequency financial time series. As far as our knowledge is concern this study is the first to highlight this equivalence on high-frequency data. We think that might be interesting to extend the analysis on more liquid markets, as the US stock market, and we plan to address this problem in the future.

References

- [1] E. Fama, *Journal of Finance*, **25**, 383-417 (2003).
- [2] A. W. Lo and A. C. MacKinlay, *Journal of Econometrics* **45**, 181-211 (1990).

- [3] J. Boudoukh, M. Richardson, and R. Whitelaw, Review of Financial Studies **7**(3), 539-573 (1994).
- [4] D. Cutler, J. Poterba, and L. Summers, Rev. of Econ. Studies **58**, 529-546 (1991).
- [5] S. Bianco and R. Renò, Journal of Futures Market, **26**, 61-84 (2006); Proc. of Spie, vol. 5848, 318-322 (2005).
- [6] A. Low and J. Muthuswamy, in *Forecasting Financial Markets: Exchange Rates, Interest rates and Asset Management*, C. Dunis ed., 3-32, John Wiley & Sons (1996).
- [7] S. Thomas and T. Patnaik, unpublished work, 2003.
- [8] C.-K. Peng, S.V. Buldyrev, S. Havlin, M. Simons, H.E. Stanley, and A.L. Goldberg, Phys. Rev. E, **49**(2):1685-1689 (1994).
- [9] J. A. O. Matos, S. M. A. Gama, H. J. Ruskin, and J. A. M. S. Duarte, Physica A, **342** 665-676 (2004).
- [10] L. Telesca, R. Lasaponara, and A. Lanorte, Physica A, **361**, 699-706 (2005).
- [11] J. Alvarez-Ramirez, G. Espinosa-Paredes, and A. Vazquez, Physica A, **351** 227-240 (2005).
- [12] R. Rizza and R. Renò, Physica A, **322**, 620-628 (2003).
- [13] M. Pasquale and R. Renò, Physica A, **346**, 518-528 (2005).